6.1 Exercise Set

1. Fill in the blank with the appropriate word or number.

a) The equation $ax^2 + bx + c = 0$ is called a quadratic equation.

b) By the zero factor property, if $(x + 2)(x - 5) = 0$, then $x = -2$ or $x = 5$.

c) The quadratic equation $(2x - 3)(x + 1) = 0$ has solutions $x =$ $3/2$ and $x =$ -1.

d) The general form of a quadratic equation is $ax^2 + bx + c = 0$

e) The equation $ab = 0$ implies $a = 0$ or $b = 0$.

f) $(3x - 2)(x + 2)$ is a quadratic function.

g) If the quadratic equation $ax^2 + bx + c = 0$ has $c = 0$, then $x = 0$ must be a solution because x is a factor.

h) The equation of the x-axis is $y = 0$.

2. Perform the indicated operations.

a) Solve $(3x - 5)(x + 3) = 0$

$x = 5/3, -3$

b) Simplify $(3x - 5)(x + 3)$

c) Solve $2(4x + 1)(x + 2) = 0$

$x = -1/4, -2$

d) Simplify $2(4x + 1)(x + 2)$

e) Solve $(1 - 3x)(2 + x) = 0$

$x = 1/3, -2$

f) Simplify $(1 - 3x)(2 + x)$

3. What went wrong?

a) $x(3x + 2) = 0$ can't ÷ by x.
$3x + 2 = 0$
$x = -\frac{2}{3}$

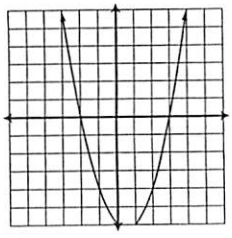
b) $x^2 = x$
$x = 1$

c) $2x^2 = 6$
$\frac{2x^2}{2} = \frac{6}{2}$
$x = \pm\sqrt{3}$

d) $x^2 - x = 6$
$x(x - 1) = 6$
$x = 6$ or $x - 1 = 6$
$x = 6$ or $x = 7$

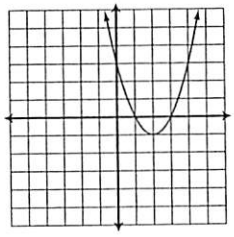
4. Use the graph of $y = f(x)$ to solve the quadratic equation $f(x) = 0$.

a) $x^2 - x - 6 = 0$

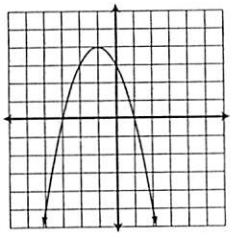


$x = -2, 3$

b) $x^2 - 4x + 3 = 0$

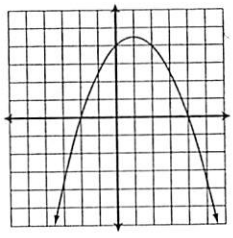


c) $-x^2 - 2x + 3 = 0$



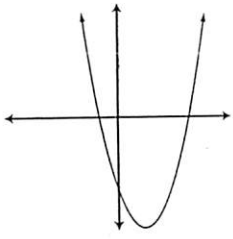
$x = -3, 1$

d) $-\frac{1}{2}x^2 + x + 4 = 0$



5. Find the x-intercepts for each quadratic equation.

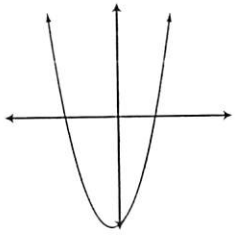
a) $y = x^2 - 3x - 4 = (x-4)(x+1)$



$x = 4, -1$

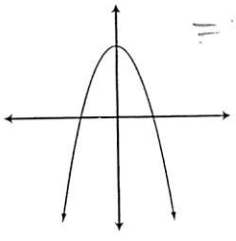
$(4,0)(-1,0)$

b) $y = x^2 + x - 6$



c) $y = -x^2 + 4 = -(x^2 - 4)$

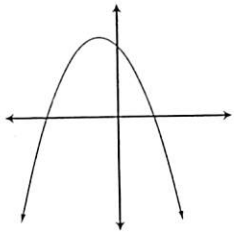
$= -(x+2)(x-2)$



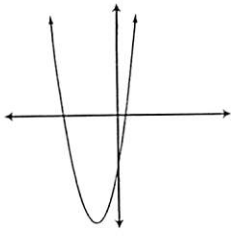
$x = \pm 2$

$(2,0)(-2,0)$

d) $y = -\frac{1}{2}x^2 - x + 4$



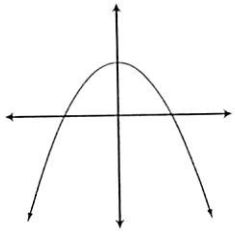
e) $y = 2x^2 + 5x - 3 = (2x-1)(x+3)$



$x = 1/2, -3$

$(1/2,0)(-3,0)$

f) $y = -\frac{1}{3}x^2 + 3$



6. Solve each equation, and check your solutions.

a) $2x(4x-3)=0$

$x=0, 3/4$

b) $\left(x-\frac{1}{3}\right)\left(3x-\frac{1}{2}\right)=0$

c) $(2y-1)(2y-1)=0$

$y=1/2$

d) $(0.25y-2)(0.2y+1)=0$

e) $z^2=81$

$z=\pm 9.$

f) $z^2=z$

g) $(x-1)(x+2)(2x-5)=0$

$x=1, -2, 5/2$

h) $x(x-2)(x-2)=0$

i) $x^2=-x$

$x^2+x=0$

$x(x+1)=0$

$x=0, -1$

j) $x^2+1=0$

k) $y^2=1$

$y^2-1=0$

$(y+1)(y-1)=0$

$y=\pm 1$

l) $4y^2=y$

m) $3x^2-x=0$

$x(3x-1)=0$

$x=0, 1/3.$

n) $\frac{2}{3}x^2=4x$

o) $\frac{x^2(x-2)}{x}=0$

$x=\not{0}, 2$

p) $x^2=3$

7. Solve each equation, and check your solutions.

a) $x^2 + 5x + 6 = 0$

$(x+3)(x+2)=0$

$x=-3,-2$

b) $x^2 - 4x + 3 = 0$

c) $y^2 + y - 12 = 0$

$(y+4)(y-3)=0$

$y=-4,3.$

d) $y^2 + 4y = -3$

e) $z(z-5) = -4$

$z^2-5z+4=0$

$(z-4)(z-1)=0$

$z=4,1$

f) $z(3z-20) = -12$

g) $(x-12)(x+1) = -40$

$x^2+x-12x-12+40=0$

$x^2-11x+28=0$

$(x-7)(x-4)=0 \quad x=4,7$

h) $(x+5)(x+3) = 5x+25$

i) $(4x-5)(x-5) = -45x$

$4x^2-20x-5x+25=-45x$

$4x^2+20x+25=0$

$(2x+5)(2x+5)=0 \quad x=-5/2$

j) $(y-6)(y+1) = -10$

k) $z^3 - 16z = 0$

$z(z^2-16)=0$

$z(z+4)(z-4)=0$

$z=\pm 4, 0$

l) $z^3 - z^2 = 6z$

m) $x^3 - 3x = 2x^2$

$x^3-2x^2-3x=0$

$x(x^2-2x-3)=0$

$x(x-3)(x+1)=0$

$x=0,3,-1$

n) $\frac{x^2}{18} + \frac{x}{6} = 1$

8. Solve each equation, and check your solutions.

a) $(2x-1)^2 = 16$

$(2x-1)^2 - 16 = 0$

$(2x-1+4)(2x-1-4) = 0$

$(2x+3)(2x-5) = 0$

$x = -3/2, \ 5/2$

b) $(3x+8)(x-1) = (x-1)(x+3)$

c) $(2y)^2 + (y+5)^2 = (2y+4)^2$

$4y^2 + y^2 + 10y + 25 - 4y^2 - 16y - 16 = 0$

$y^2 - 6y + 9 = 0$

$(y-3)(y-3) = 0 \quad y = 3$

d) $(y+3)^2 = (2y-1)^2$

e) $2z(z-\sqrt{2}) = -1$

$2z^2 - 2\sqrt{2}z + 1 = 0$

$(\sqrt{2}z - 1)(\sqrt{2}z - 1) = 0$

$z = 1/\sqrt{2} = \sqrt{2}/2$

f) $(z-2)^2 + (z-1)^2 = 5$

g) $6x^2(3x-1) - x(3x-1) = 2(3x-1)$ (a, a, a)

$a6x^2 - ax - 2a = 0$

$a(6x^2 - x - 2) = 0$

$a(3x-2)(2x+1) = 0$

$(3x-1)(3x-2)(2x+1) = 0$

$x = 1/3, \ 2/3, \ -1/2$

h) $\dfrac{x-4}{x-3} + \dfrac{x-2}{x-3} = x-3$

i) $\left[\dfrac{1}{x} + \dfrac{3}{x-2} = \dfrac{5}{8}\right] 8x(x-2)$

$8(x-2) + 3(8x) = 5x(x-2)$

$8x - 16 + 24x = 5x^2 - 10x$

$0 = 5x^2 - 42x + 16$

$0 = (5x-2)(x-8) \quad x = 2/5, \ 8$

j) $\dfrac{4}{5} + y = \dfrac{4y-50}{5y-25}$

k) $\left[\dfrac{4}{x^2-4} - \dfrac{1}{x-2} = 1\right] (x+2)(x-2)$

$(x+2)(x-2)$

$4 - (x+2) = x^2 - 4$

$0 = x^2 + x - 6 \quad x = -3, \ \cancel{2}$

$0 = (x+3)(x-2)$

l) $\dfrac{1}{x-3} - \dfrac{12}{x^2-9} = 1$

9. Write a quadratic equation with the given solutions.

a) 0, 3 $x(x-3)=0$ $x^2-3x=0$

b) $-2, 2$ $(x+2)(x-2)=0$

c) $-\frac{2}{3}, 4$ $(3x+2)(x-4)=0$

d) $-\frac{3}{4}, \frac{1}{3}$ $(4x+3)(3x-1)=0$

e) -3 $(x+3)^2=0$

f) $\sqrt{2}$ $(x-\sqrt{2})^2=0$

10. The length of a rectangle is three cm less than twice the width. Find the dimensions of the rectangle if the area is 20 cm².

$w(2w-3)=20$

$2w^2-3w-20=0$

$(2w+5)(w-4)=0$

$w=-\frac{5}{2}, 4$ cm

$l=5$ cm

11. The length of a rectangle is one more than five times the width. Find the dimensions of the rectangle, if the area is 130 cm².

12. The base of a triangle is 3 m longer than the height. Find the height, if the area of the triangle is 35 m².

$\frac{h(h+3)}{2}=35$

$h(h+3)=70$

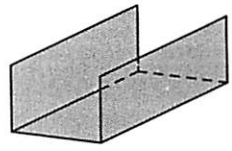
$h^2+3h-70=0$

$(h+10)(h-7)=0$

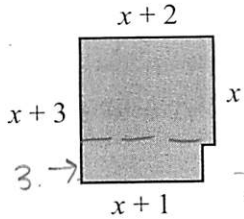
$h=-10, 7$

height = 7m.

13. The three sides of a trough are 30 cm wide in total and the cross-sectional area is 112 cm². Find the height of the trough.



14. Find x, if the shaded area is 129 cm².



$(x+2)(x)+3(x+1)=129$

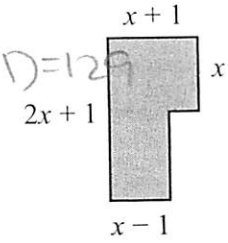
$x^2+2x+3x+3=129$

$x^2+5x-126=0$

$(x+14)(x-9)=0$

$x=-14, 9$ $x=9$ cm.

15. Find x, if the shaded area is 54 cm².



6.2 Exercise Set

1. Fill in the blanks to make the statement true.

a) $ax^2 + bx + c = 0$ is the general quadratic equation

b) $x^2 = k,\ k > 0$ has 2 solutions of $\sqrt{k}$ and $-\sqrt{k}$.

c) $x^2 = 0$ has 1 solution of zero.

d) $x^2 = -k,\ k > 0$ has no solutions.

2. Solve. Give the exact solutions.

a) $\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

b) $x^2 = -16$

c) $2x^2 = 11$

$x^2 = 11/2$

$x = \pm\sqrt{11/2}$

d) $(x-1)^2 = 9$

e) $(2x+1)^2 = 7$

$2x+1 = \pm\sqrt{7}$

$2x = -1 \pm \sqrt{7}$ $\quad x = \dfrac{-1 \pm \sqrt{7}}{2}$

f) $(3x-1)^2 = 0$

g) $(x-3)^2 = 3$

$x-3 = \pm\sqrt{3}$

$x = 3 \pm \sqrt{3}$

h) $(x-2)^2 = -2$

i) $6x^2 = 4x^2 + 5$

$2x^2 = 5$.

$x^2 = 5/2$ $\quad x = \pm\sqrt{5/2}$

j) $(x-2)^2 = 8$

k) $\left(x + \frac{2}{3}\right)^2 = \frac{2}{9}$ $\quad x + \frac{2}{3} = \pm\frac{\sqrt{2}}{3}$

$x = -\dfrac{2 \pm \sqrt{2}}{3}$

l) $\left(x - \frac{3}{8}\right)^2 = \frac{7}{16}$

3. Find the perfect square trinomial whose first two terms are given.

a) $x^2 + 4x + 4 = (x+2)^2$

b) $y^2 - 2y$

c) $z^2 + 3z + \frac{9}{4} = (x+\frac{3}{2})^2$

d) $x^2 - \frac{3}{2}x$

e) $y^2 + \frac{2}{3}y + \frac{4}{36} \to \frac{1}{9} = (y+\frac{1}{3})^2$ ↳ $\frac{2}{6}$

f) $z^2 - \frac{6}{5}z$

4. Factor each perfect square trinomial.

a) $x^2 + 6x + 9$
$(x+3)^2$

b) $y^2 - 10y + 25$

c) $z^2 - 3z + \frac{9}{4}$
$(z - \frac{3}{2})^2$

d) $x^2 - \frac{6}{5}x + \frac{9}{25}$

e) $y^2 + \frac{8}{9}x + \frac{16}{81}$
$(y+\frac{4}{9})^2$

f) $z^2 - \frac{3}{2}z + \frac{9}{16}$

5. Solve by factoring the perfect square trinomial.

a) $x^2 + 6x + 9 = 1$
$(x+3)^2 = 1$
$x+3 = \pm 1$
$x = -3 \pm 1$
$x = -2, -4$

b) $y^2 - 4y + 4 = 9$

c) $z^2 + \frac{10}{3}z + \frac{25}{9} = \frac{8}{9}$
$(z+\frac{5}{3})^2 = \frac{8}{9}$
$z + \frac{5}{3} = \pm \frac{2\sqrt{2}}{3}$
$z = \frac{-5 \pm 2\sqrt{2}}{3}$

d) $x^2 - x + \frac{1}{4} = \frac{5}{2}$

e) $y^2 + \frac{2}{3}y + \frac{1}{9} = \frac{4}{9}$
$(y+\frac{1}{3})^2 = \frac{4}{9}$
$y + \frac{1}{3} = \pm \frac{2}{3}$
$y = -\frac{1}{3} \pm \frac{2}{3}$ → $\frac{1}{3}, -1$

f) $z^2 - 0.6z + 0.09 = 0.64$

6. Solve the quadratic equation by completing the square. Use exact answers when possible.

a) $x^2 + 6x = -5$

$(x+3)^2 - 9 = -5$

$(x+3)^2 = 4$

$x+3 = \pm 2$

$x = -3 \pm 2$ $\quad x = -5, -1$

b) $y^2 - 5y + 3 = 0$

c) $z^2 - 8z + 3 = 0$

$(z-4)^2 - 16 + 3 = 0$

$(z-4)^2 = 13.$

$z - 4 = \pm\sqrt{13}$ $\quad z = 4 \pm \sqrt{13}.$

d) $x^2 - 2x - 1 = 0$

e) $y^2 + 4y + 2 = 0$

$(y+2)^2 - 4 + 2 = 0$

$(y+2)^2 = 2$

$y + 2 = \pm\sqrt{2}$ $\quad y = -2 \pm \sqrt{2}$

f) $z^2 + 2z + 7 = 0$

g) $x^2 - x - 8 = 0$

$(x - 1/2)^2 - 1/4 - 8 = 0$

$(x - 1/2)^2 - 33/4 = 0.$

$x - 1/2 = \pm\frac{\sqrt{33}}{2}$ $\quad x = \frac{1}{2} \pm \frac{\sqrt{33}}{2}$

h) $y(y-2) = 7$

i) $z(z+5) = -2$

$z^2 + 5z + 2 = 0$

$(z + 5/2)^2 - 25/4 + 2 = 0$

$(z + 5/2)^2 - \frac{17}{4} = 0$ $\quad z + 5/2 = \pm\frac{\sqrt{17}}{2}$ $\quad z = -\frac{5 \pm \sqrt{17}}{2}$

j) $x(3-x) = 4$

k) $x(2-x) = -1$

$-x^2 + 2x + 1 = 0$

$-(x^2 - 2x + 1 - 1) + 1 = 0$

$-(x-1)^2 + 2 = 0$ $\quad -(x-1)^2 = +2$

$x - 1 = \pm\sqrt{2}$ $\quad x = 1 \pm \sqrt{2}$

l) $(1-x)(2+x) = -1$

7. Solve the quadratic equation by completing the square. Use exact answers.

a) $2x^2 + 4x - 1 = 0$

$2(x^2+2x+1-1)-1=0$

$2(x+1)^2-2-1=0$

$2(x+1)^2=3$

$x+1=\pm\sqrt{\frac{3}{2}}$ $\quad x=-1\pm\sqrt{3}/2$

b) $2x^2 + x = 5$

c) $5x^2 - 3x = 1$

$5(x^2-\frac{3}{5}x+\frac{9}{100}-\frac{9}{100})=1$

$5(x-3/10)^2-\frac{45}{100}=1$

$5(x-3/10)^2=\frac{145}{100}$

$(x-3/10)^2=\frac{145}{500}=\frac{29}{100}$

$x-\frac{3}{10}=\pm\frac{\sqrt{29}}{10}$

$x=\frac{3}{10}\pm\frac{\sqrt{29}}{10}$

d) $3x^2 - x = 3$

e) $3x^2 = -8x - 2$

$3x^2+8x=-2$

$3(x^2+\frac{8}{3}x+\frac{16}{9}-\frac{16}{9})=-2$

$3(x+\frac{4}{3})^2-\frac{16}{3}=-2$

$3(x+4/3)^2=\frac{10}{3}$

$(x+4/3)^2=\frac{10}{9}$

$(x+4/3)=\pm\sqrt{10}/3$ $\quad x=\frac{-4\pm\sqrt{10}}{3}$

f) $3x^2 = 6x + 2$

g) $2x^2 - 2\sqrt{2}x = -1$

$2(x^2-\sqrt{2}x+\frac{1}{2}-\frac{1}{2})=-1$

$\sqrt{2}/2$, $(\sqrt{2}/2)^2$

$2(x-\sqrt{2}/2)^2-1=-1$

$2(x-\sqrt{2}/2)^2=0$ $\quad x=\frac{\sqrt{2}}{2}$

h) $-9x^2 + 18x - 3 = 0$

i) $-9x^2 + 12x + 14 = 0$

$-9(x^2-\frac{4}{3}x+\frac{4}{9}-\frac{4}{9})=14$

$-9(x-2/3)^2+4=-14$

$(x-2/3)^2=\frac{-18}{-9}$ $\quad x-\frac{2}{3}=\pm\sqrt{2}$

$x=\frac{2}{3}\pm\sqrt{2}$

j) $-4x^2 + 4x + 99 = 0$

6.3 Exercise Set

1. Simplify the expressions. Leave answers in radical form.

a) $\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-8)}}{2\cdot 1}$

b) $\frac{-(-3)\pm\sqrt{(-3)^2-4(-2)(2)}}{2(-2)}$

$\frac{4\pm\sqrt{48}}{2} = \frac{4\pm 4\sqrt{3}}{2} = 2\pm 2\sqrt{3}$.

c) $\frac{-(-2)\pm\sqrt{(-2)^2-4(\sqrt{3})(-\sqrt{3})}}{2\sqrt{3}}$

d) $\frac{\sqrt{2}\pm\sqrt{(\sqrt{2})^2-4(2)(-2)}}{2\cdot 2}$

$\frac{2\pm\sqrt{4+12}}{2\sqrt{3}} = \frac{2\pm\sqrt{16}}{2\sqrt{3}} = \frac{2\pm 4}{2\sqrt{3}} = \frac{1\pm 2}{\sqrt{3}} = \frac{\sqrt{3}\pm 2\sqrt{3}}{3} = -\frac{\sqrt{3}}{3}, \sqrt{3}$

e) $\frac{-(-\sqrt{3})\pm\sqrt{(-\sqrt{3})^2-4(\frac{1}{2})(2)}}{2(\frac{1}{2})}$

f) $\frac{-(-\sqrt{6})\pm\sqrt{(-\sqrt{6})^2-4(3)(\frac{1}{2})}}{2(3)}$

$\frac{\sqrt{3}\pm\sqrt{3-4}}{1}$ $\quad\varnothing$.

2. Write the equation in standard form and identify the values of a, b, and c.

a) $6-2x^2=3x$

$0 = 2x^2+3x-6$

$a \quad b \quad c$

b) $2+\frac{x^2}{3}=\frac{5x}{4}$

c) $\left[\frac{x^2}{2}=\frac{4x}{3}\right]6$

$3x^2 = 8x$

$3x^2-8x=0$

$a \quad b \quad c=0$

d) $\frac{2x^2}{3}+1=\frac{x}{5}$

e) $x(3x-5)+7=5x^2-2x$

$3x^2-5x+7=5x^2-2x$

$0=2x^2+3x-7$

$a \quad b \quad c$

f) $6x(x+1)=(x+3)^2$

3. Solve the equation using the quadratic formula. Leave answers in radical form.

a) $x^2 = -4x - 1$

$x^2 + 4x + 1 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(1)}}{2(1)} = \frac{-4 \pm \sqrt{12}}{2} =$

$\frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$

b) $x^2 = 4 - 4x$

c) $x^2 = -4x + 1$

$x^2 + 4x - 1 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(-1)}}{2(1)} = \frac{-4 \pm \sqrt{20}}{2}$

$= \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$

d) $2x^2 = 3x + 1$

e) $\left[\frac{x^2}{4} + \frac{1}{8} = \frac{x}{2}\right]^8$

$2x^2 + 1 = 4x \quad 2x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16 - 4(2)(1)}}{2(2)} = \frac{4 \pm \sqrt{8}}{4} = \frac{4 \pm 2\sqrt{2}}{4}$

$= \frac{2 \pm \sqrt{2}}{2}$

f) $8x^2 - 20x - 3 = 0$

g) $\left[\frac{x(2x+1)}{x-2} = \frac{10}{x-2}\right](x-2)$

h) $(x-2)(x+4) = 2x(x-3)$

$x(2x+1) = 10$

$2x^2 + x - 10 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4(2)(-10)}}{2(2)} = \frac{-1 \pm \sqrt{81}}{4} = \frac{-1 \pm 9}{4} = 2, -5/2.$

i) $\left[\frac{x^2}{12} + \frac{x}{4} = -\frac{1}{3}\right]^{12}$

j) $(x+3)^2 = 6x(x+1)$

$x^2 + 3x = -4$

$x^2 + 3x + 4 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4(4)}}{2(1)}$ $\emptyset$

4. Solve each of the following equations using the quadratic formula. Use exact answers when possible.

a) $y^2 - 4y - 3 = 0$

$y = \frac{4 \pm \sqrt{16 - 4(-3)}}{2} = \frac{4 \pm \sqrt{28}}{2}$

$= \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}$

b) $x^2 + 5x - 3 = 0$

c) $3x^2 - 6x + 4 = 0$

$x = \frac{6 \pm \sqrt{36 - 4(3)(4)}}{2(3)}$ $\varnothing$

d) $\frac{3}{2}y^2 - 2 = y$

e) $5z(z+2) = -4$

$5z^2 + 10z + 4 = 0$

$z = \frac{-10 \pm \sqrt{100 - 4(5)(4)}}{2(5)} = \frac{-10 \pm \sqrt{20}}{10}$

$= \frac{-10 \pm 2\sqrt{5}}{10} = \frac{-5 \pm \sqrt{5}}{5}$

f) $12x = 3x^2 - 8$

g) $(x-2)(x-3) = 8$

$x^2 - 5x + 6 = 8$

$x^2 - 5x - 2 = 0$

$x = \frac{5 \pm \sqrt{25 - 4(-2)}}{2} = \frac{5 \pm \sqrt{33}}{2}$

h) $\frac{2}{x+2} + \frac{1}{x} = 1$

i) $\left[\frac{2}{y} = \frac{3}{y^2} + 2\right] y^2$

$2y = 3 + 2y^2$

$0 = 2y^2 - 2y + 3$

$y = \frac{2 \pm \sqrt{4 - 4(2)(3)}}{2(2)}$ $\varnothing$

j) $\frac{1}{x} - \frac{2}{x-4} = 2$

k) $5.13x^2 - 7.27x - 4.32 = 0$

$x = \frac{7.27 \pm \sqrt{(-7.27)^2 - 4(5.13)(-4.32)}}{2(5.13)}$

$= \frac{7.27 \pm \sqrt{141.4993}}{10.26} = 1.868, -0.451$

l) $\sqrt{3}x^2 = 8\sqrt{2}x - 4\sqrt{3}$

5. Solve the equation by any method.

a) $\left[\frac{x}{3} - 2 = -\frac{3}{x}\right] 3x$

$x^2 - 6x = -9$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3.$

b) $\frac{2x}{3x-1} = \frac{2x-3}{x+1}$

c) $\left[\frac{x+2}{x} + \frac{x}{x-2} = 5\right] x(x-2)$

$(x+2)(x-2) + x^2 = 5x(x-2)$

$x^2 - 4 + x^2 = 5x^2 - 10x$

$0 = 3x^2 - 10x + 4$

$x = \frac{10 \pm \sqrt{100 - 4(3)(4)}}{2(3)} = \frac{10 \pm \sqrt{52}}{6}$

$= \frac{10 \pm 2\sqrt{13}}{6} = \frac{5 \pm \sqrt{13}}{3}$

d) $\frac{2}{x+4} - \frac{3}{x+1} = 4$

6. Solve the equation by any method.

a) $\left(\sqrt{2x-1}\right)^2 = (x-2)^2$

$2x - 1 = x^2 - 4x + 4$

$0 = x^2 - 6x + 5$

$0 = (x-5)(x-1)$

$x = 5, \cancel{x}.$

b) $\sqrt{x^2+1} = \sqrt{3x+2}$

c) $\left(\sqrt{2-3y}\right)^2 = (y+1)^2$

$2 - 3y = y^2 + 2y + 1$

$0 = y^2 + 5y - 1$

$y = \frac{-5 \pm \sqrt{25 - 4(-1)}}{2} = \frac{-5 \pm \sqrt{29}}{2}$

d) $\sqrt{x+3} = 1 - x$

7. Solve the equation by any method.

a) $x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2-1)=0$

$(x+2)(x-2)(x+1)(x-1)=0$

$x = \pm 2, \pm 1$

b) $x^4 - 5x^3 - 36 = 0$

c) $6x^4 - 13x^2 + 5 = 0$

$(3x^2-5)(2x^2-1)=0$

$x^2 = 5/3 \quad x = \pm\sqrt{5/3}$

$x^2 = 1/2 \quad x = \pm\sqrt{1/2}$

d) $4x^4 - x^2 - 20 = 0$

e) $\left(\frac{x^2+2}{x}\right)^2 - 6\left(\frac{x^2+2}{x}\right) + 5 = 0$

$a = 5, 1$

$a^2 - 6a + 5 = 0 \quad (a-5)(a-1)=0$

$\frac{x^2+2}{x} = 5 \quad x^2+2 = 5x \quad x^2-5x+2=0 \quad x = \frac{5 \pm \sqrt{25-4(2)}}{2} = \frac{5 \pm \sqrt{17}}{2}$

$\frac{x^2+2}{x} = 1 \quad x^2+2 = x \quad x^2-x+2=0 \quad x = \frac{1 \pm \sqrt{1-4(2)}}{2} \quad \varnothing$

f) $\left(\frac{x^2+1}{x}\right)^2 + 4\left(\frac{x^2+1}{x}\right) - 12 = 0$

8. Locate the first error in the working.

a) $x^2 - 4x + 1 = 0$

$$x = \frac{-(-4) \pm \sqrt{16 - 4(1)(1)}}{2}$$

$$= \frac{-4 \pm \sqrt{12}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= -2 \pm \sqrt{3}$$

b) $x^2 - 4x - 2 = 0$

$$x = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$= 2 \pm \sqrt{2}$$

c) $x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$= 4 \pm \frac{\sqrt{8}}{2}$$

$$= 4 \pm \frac{2\sqrt{2}}{2}$$

$$= 4 \pm \sqrt{2}$$

d) $x^2 - 4x - 2 = 0$

$$x = \frac{4 \pm \sqrt{16 + 8}}{2}$$

$$= \frac{4 \pm \sqrt{24}}{2}$$

$$= \frac{4 \pm 2\sqrt{6}}{2}$$

$$= 4 \pm \sqrt{6}$$

2. Find the discriminant D, and use it to determine the number and kind of roots of each equation.

a) $x^2 - 8x + 16 = 0$

$D = 64 - 4(16) = 0$

1 real root

b) $2x^2 - x - 3 = 0$

c) $3x^2 - 4x + 5 = 0$

$D = 16 - 4(3)(5) < 0$

no real roots.

d) $3x^2 - 5x + 1 = 0$

e) $2x^2 - 6x = 0$

$D = 36 - 4(2) > 0$

2 real roots

f) $\frac{3x-2}{2x-1} = \frac{1}{2-x}$

g) $(2x + 3)(x - 1) = x + 5$

$2x^2 - 2x + 3x - 3 = x + 5$

$2x^2 - 8 = 0$.

$D = 0 - 4(2)(-8) > 0$

2 real roots.

h) $x^2 - 2\sqrt{2}x + 2 = 0$

i) $\sqrt{3}x^2 - 6x + 3\sqrt{3} = 0$

$D = 36 - 4(\sqrt{3})(3\sqrt{3}) = 0$

1 real root.

j) $-\sqrt{2}x^2 + 3x + 2\sqrt{2} = 0$

3. Use the value of the discriminant D to determine if the trinomial can be factored.

a) $24x^2 + 7x - 5 = 0$

$D = 49 - 4(24)(-5) = 529$

↗

perfect square

so yes

b) $11x^2 - 9x + 2 = 0$

c) $2x^2 - 2\sqrt{6}x + 3 = 0$

$D = (-2\sqrt{6})^2 - 4(2)(3)$

$4(6) - 24 = 0$

yes.

d) $6x^2 + x - 8 = 0$

4. Determine the values of k for which the graph of the function will have the indicated number of crossings of the x-axis.

a) $f(x) = x^2 + kx + 6$; one crossing

$k^2 - 4(6) = 0$

$k^2 = 24$

$k = \pm 2\sqrt{6}$

b) $f(x) = x^2 + 6x + k$; zero crossings

c) $f(x) = x^2 - 6x + k$; two crossings

$36 - 4(k) > 0$

$36 > 4k$

$9 > k$.

d) $f(x) = kx^2 - 40x - 25$; one crossing

5. Determine the value of k so that the equation has the indicated number of solutions.

a) $kx^2 + x + k = 0$; one real solution

$1 - 4(k)(k) = 0$.

$1 = 4k^2$

$\frac{1}{4} = k^2$

$k = \pm \frac{1}{2}$

b) $kx^2 + x + k = 0$; two real solutions

c) $kx^2 + x + k = 0$; no real solutions

$1 - 4(k)(k) < 0$

$1 < 4k^2$

$\frac{1}{4} < k^2$ $k > \frac{1}{2}$ or $k < -\frac{1}{2}$

d) $x^2 - kx + 4 = 0$; one real solution

e) $x^2 - kx + 4 = 0$; two real solutions

$k^2 - 4(4) > 0$

$k^2 > 16$

$k > 4$ $k < -4$

f) $x^2 - kx + 4 = 0$; no real solutions

6.5 Exercise Set

1. Find two consecutive odd whole numbers so that the sum of their squares is 130.

$x^2 + (x+2)^2 = 130$

$x^2 + x^2 + 4x + 4 = 130$

$2x^2 + 4x - 126 = 0$

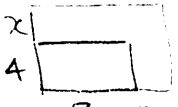
$x^2 + 2x - 63 = 0$

$(x+9)(x-7) = 0$

$x = -9, 7$ $\boxed{7, 9}$

2. The sum of a number and three times its reciprocal is 4. What numbers have this property?

3. The length and width of a rectangular sheet of plywood is 4 ft by 8 ft. How much must be added equally to the length and width to double the area?



$(4+x)(8+x) = 64$

$32 + 12x + x^2 = 64$

$A = 32$

$x^2 + 12x - 32 = 0$

$x = \frac{-12 \pm \sqrt{144 - 4(-32)}}{2}$

$x = 2.25$ ft

4. Two planes travel at right angles to each other after leaving an airport at the same time. After one hour the planes are 390 km apart. If one plane travels 210 km/h faster than the other, what is the speed of the slower plane?

5. A boat takes 1 hr longer to go 36 km up a river than to go down the river. If the boat travels 15 km/h in still water, what is the speed of the current?

	S	T	D
up	$15-c$	$\frac{36}{15-c}$	36
down	$15+c$	$\frac{36}{15+c}$	36

$\left[\frac{36}{15-c} = \frac{36}{15+c} + 1\right](15-c)(15+c)$

$36(15+c) = 36(15-c) + 225 - c^2$

$540 + 36c = 540 - 36c + 225 - c^2$

$c^2 + 72c - 225 = 0$

$(c+75)(c-3) = 0$

$c = -75, 3$ km/h

6. The total surface area of a square base box with a height of 9 cm is 350 cm². Find the length of the base of the box.

7. John takes 4 hours to weed the garden alone, and Mike takes 6 hours to do the same job. How long does it take them to do the job together?

$$\left[\frac{x}{4}+\frac{x}{6}=1\right]^{24}$$

$6x+4x=24$

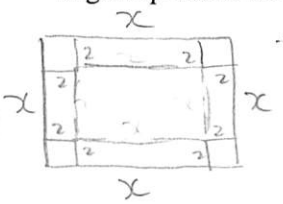
$10x-24=0$

$x=24/10=2.4$ hrs.

or 2 hrs 24 min.

8. A ranch uses 200 m of fencing to enclose two adjacent rectangular corrals. Find the dimensions that enclose a total area of 1400 m².

9. From each corner of a square piece of cardboard, a square with sides of 2 cm is removed. The edges are then turned up to form an open box. If the box is to hold 200 cm³, what are the dimensions of the original piece of cardboard?



$2(x-4)(x-4)=200$

$(x-4)^2=100$

$x^2-8x+16=100$

$x^2-8x-84=0$

$(x-14)(x+6)=0$

$x=14, \cancel{-6}$

14 cm × 14 cm

10. A rectangle has an area of 72 m². If the width is increased by 2 m and the length is decreased by 3 m, the area stays the same. Find the original length and width of the rectangle.

11. James drives 300 km. If he increases his speed by 10 km/h, it takes 1 hour less time. Find James's original speed.

S	T	D.
x	$\frac{300}{x}$	300
$x+10$	$\frac{300}{x+10}$	300

$$\left[\frac{300}{x}=\frac{300}{x+10}+1\right]^{x(x+10)}$$

$300(x+10)=300x+x(x+10)$

$300x+3000=300x+x^2+10x$

$x^2+10x-3000=0$

$(x+60)(x-50)=0$

$x=-\cancel{60}, 50$ km/h

12. A circular lawn is surrounded by a flower bed of uniform width. If the flower bed has an area of 36 m² and the radius of the entire garden is 8 m, find the width of the flower bed.

13. A carpenter and his helper can do a job in 4 hours. Working alone, the carpenter can do the job in 6 hours. How long would it take the helper to do the job alone?

$$\left[\frac{4}{6} + \frac{4}{x} = 1\right]^{6x}$$

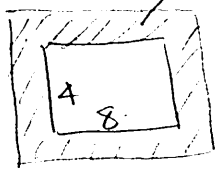
$$4x + 24 = 6x$$

$$24 = 2x$$

$$12\text{ hrs} = x$$

14. The rate of a stream is 4 km/h. Scott paddles downstream a distance of 9 km and then back upstream to where he started. If the total trip took 10 hours, what is Scott's rate of paddling in still water?

15. A gardener surrounds a 4 m × 8 m rectangular flower bed with a border of mulch of uniform width. If there is enough mulch to cover 28 m², how wide is the border?



outside − inside

$$(2x+8)(2x+4) - 32 = 28$$

$$4x^2 + 24x + 32 - 32 = 28$$

$$4x^2 + 24x - 28 = 0$$

$$x^2 + 6x - 7 = 0$$

$$(x+7)(x-1) = 0$$

$x = -7$, 1 m.

16. A playing field measures 120 m by 240 m. Karen wants to mow two-thirds of the field by mowing around the outside of the field. How wide of a strip does she mow?

17. Matt paddles 5 km/h in still water. It takes him 1 hour longer to paddle 12 km upstream than to make the return trip downstream. Find the speed of the current.

	S	T	D
up	$5-c$	$\frac{12}{5-c}$	12
down	$5+c$	$\frac{12}{5+c}$	12

$$\left[\frac{12}{5-c} - 1 = \frac{12}{5+c}\right]^{(5-c)(5+c)}$$

$$12(5+c) - (25-c^2) = 12(5-c)$$

$$60 + 12c - 25 + c^2 = 60 - 12c$$

$$c^2 + 24c - 25 = 0$$

$$(c+25)(c-1) = 0$$

$c = -25$, 1 km/h.

18. The standard running track size for track and field events is 400 m. The track consists of two semi-circles connected by parallel straight lanes. If the infield of the track encloses an area of 9430 m², find the length of the straight lanes, and the diameter of the track.

6.6 Chapter Review

Section 6.1

1. Solve.

a) $4x^2 - x = 39$ $4x^2-x-39=0$

$(4x-13)(x+3)=0$ $x=13/4, -3$

b) $3y^2 + y = 24$

c) $3z^2 + 3 = -10z$ $3z^2+10z+3=0$

$(3z+1)(z+3)=0$

$z=-1/3, -3$

d) $2x^2 - 12 - 4x = x^2 - 3x$

e) $9y^2 = 6y - 1$ $9y^2-6y+1=0$

$(3y-1)(3y-1)=0$ $y=1/3$

f) $(5z + 1)(z + 3) = -2(5z + 1)$

g) $(3x + 1)(x - 3) = 2 + 3(x + 5)$

$3x^2-8x-3=2+3x+15$

$3x^2-11x-20=0$ $(3x+4)(x-5)=0$

$x=-4/3, 5$

h) $(2y + 1)(y + 1) = 2(1 - y) + 6$

i) $(x - 3)(3x + 4) = (x + 2)(x - 6)$ $3x^2-5x-12=x^2-4x-12$

$2x^2-x=0$

$x(2x-1)=0$ $x=0, 1/2$

j) $(2x - 3)(x + 6) = (x - 9)(x + 2)$

2. Write a quadratic equation with the given solutions.

a) $2, \frac{1}{3}$ $(x-2)(3x-1)=0$

b) -4

Section 6.2

3. Solve by the square root property.

a) $\left(x + \frac{2}{3}\right)^2 = \frac{5}{9}$ $x+\frac{2}{3}=\pm\frac{\sqrt{5}}{3}$

$x=\frac{-2\pm\sqrt{5}}{3}$

b) $\left(y - \frac{3}{2}\right)^2 = \frac{7}{4}$

c) $(2x - 1)^2 = -1$ $2x-1=\pm\sqrt{-1}$ $\emptyset$

d) $(1 - 3x)^2 = 0$

4. Find the perfect square trinomial whose first two terms are given.

a) $x^2 - \frac{4}{3}x + \frac{4}{9} = \left(x - \frac{2}{3}\right)^2$ ↳ $\frac{4}{6} \div 2 = \frac{2}{3}$

b) $y^2 + \frac{1}{4}y$

c) $z^2 - \frac{3}{5}z + \frac{9}{100} = \left(z - \frac{3}{10}\right)^2$ ↳ $\frac{3}{10}$

d) $x^2 + \frac{2}{3}x$

5. Solve by completing the square.

a) $2x^2 + x - 2 = 0$

$2\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) = 2$

$2\left(x + \frac{1}{4}\right)^2 - \frac{1}{8} = 2 \quad 2\left(x + \frac{1}{4}\right)^2 = \frac{17}{8}$

b) $2t^2 + 3t - 4 = 2t - 1$

$x + \frac{1}{4} = \pm\frac{\sqrt{17}}{4}$

$x = \frac{-1 \pm \sqrt{17}}{4}$

c) $(x - 2)(x + 1) = 6$

$x^2 - x - 2 = 6$

$x^2 - x - 8 = 0$

$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 8 = 0$

d) $(2x + 5)(x - 3) = (x + 4)(x - 1)$

$\left(x + \frac{1}{2}\right)^2 = \frac{33}{4}$

$x + \frac{1}{2} = \pm\frac{\sqrt{33}}{2}$

$x = \frac{-1 \pm \sqrt{33}}{2}$

Section 6.3

6. Solve by the quadratic formula.

a) $x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

b) $-2x^2 + 2x + 1 = 0$

c) $2x^2 + x + 2 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4(2)(2)}}{2(2)}$ ∅

d) $2x^2 + 4x - 5 = 0$

e) $\left[\frac{2}{x} + \frac{3}{x - 1} = 1\right]^{x(x-1)}$

$2(x - 1) + 3x = x(x - 1)$

$2x - 2 + 3x = x^2 - x$

$0 = x^2 - 6x + 2$

$x = \frac{6 \pm \sqrt{36 - 4(2)}}{2} = \frac{6 \pm \sqrt{28}}{2} = \frac{6 \pm 2\sqrt{7}}{2}$

$= 3 \pm \sqrt{7}$

f) $\frac{6}{x} + \frac{4}{x + 2} = 1$

g) $(x - 1)(x + 2) = 1$

$x^2 + x - 2 = 1$

$x^2 + x - 3 = 0$

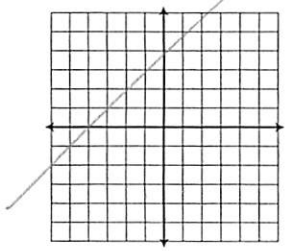
$x = \frac{-1 \pm \sqrt{1 - 4(-3)}}{2} = \frac{-1 \pm \sqrt{13}}{2}$

h) $(x + 1)(x - 2) = 2$

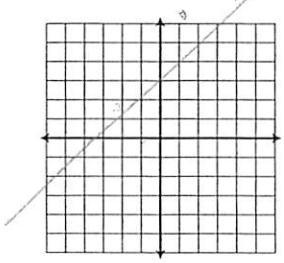
Section 6.4

7. Solve with a graphing calculator. Make a sketch of the solution.

a) $-\frac{1}{2}x^2 - 2x + 3 = 0$



b) $-x^2 - 6x = 10$



8. Calculate the discriminant D, then determine the number and kind of roots of each equation.

a) $f(x) = x^2 - 2x - 1$

$D = 4 - 4(-1) > 0 \quad \therefore$ 2 real roots.

b) $f(x) = 2x^2 + x + 1$

c) $f(x) = 2x^2 - 2\sqrt{2}x + 1$

$D = (-2\sqrt{2})^2 - 4(2)(1)$

$4(2) - 8 = 0 \quad \therefore$ 1 real root.

d) $f(x) = 2x^2 + x - 6$

9. Determine the value of k, so that the equation has the indicated number of solutions.

a) $kx^2 + x + k = 0$; two real solutions

$1 - 4(k)(k) > 0 \quad \frac{1}{4} > k^2$

$1 > 4k^2 \quad -\frac{1}{2} < k < \frac{1}{2}$

b) $2x^2 + kx + 2 = 0$; one real solution

c) $x^2 - kx + 1 = 0$; no real solutions

$k^2 - 4(1) < 0$

$k^2 < 4 \quad -2 < k < 2$

d) $3x^2 - kx + k = 0$; one real solution.

Section 6.5

10. Vancouver and Kamloops are 300 km apart. Jerry travels 20 km/h faster than Carol. Find Jerry's average speed if it takes him 1.25 hrs less time to travel from Vancouver to Kamloops than Carol.

	S	T	D
J	$x+20$	$\frac{300}{x+20}$	300
C	x	$\frac{300}{x}$	300

$\left[\frac{300}{x+20} + 1.25 = \frac{300}{x}\right] x(x+20)$

$300x + 1.25x(x+20) = 300(x+20)$

$300x + 1.25x^2 + 25x = 300x + 6000$

$1.25x^2 + 25x - 6000 = 0.$

$5x^2 + 100x - 24000 = 0$

$(5x + 400)(x - 60) = 0$

$x = 60$ km/h $\quad J = 80$ km/h

11. Two pipes together can fill a tank in 2 hours. One of the pipes used alone takes 3 hours longer than the other to fill the tank. How long does it take the slower pipe to fill the tank alone?